AN ANALITICAL METHOD FOR INTERSECTION ENTERNING TRAFFIC FLOW ESTIMATION

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Abstract

In this paper, we try to propose a simple but analytical approach to describe distribution of intersection entering traffics on a grid road network. The approach is a continuous model intersection oriented and depending on the network parameters only. This approach has an explicit structure and free of the need of a detailed *OD* (Origin-Destination) table thus very useful for the planning stage of road network. Using a proved commercial, the proposed approach is finally validated as of acceptable accuracy.

1. Introduction

For the purpose of proper road network planning, it is very important to predict the traffic distribution on a road network. Currently the most widely used approaches can be roughly divided into two types: the static traffic assignment models and the dynamic traffic simulation (macroscopic or microscopic) models. However, the traditional models inevitably require detailed *OD* information, and complicated black-box type of algorithms. As a result, users often reach to their computation outputs without knowing the underlying logic behind. Furthermore, although there are some exceptions, the traditional models are all link oriented while intersections play a trivial role (if any) in the traditional models, despite the fact that it is the intersections rather than the basic sections of road links, decide the performance of urban road networks in modern cities.

In this paper we assume the passing probability of an intersection equals to the utilization of the intersection among all routes between an *OD* pair within a road network. Under this assumption, an explicit analytical approach is developed to estimate the distribution of traffic within a grid network from a single pair of *OD*. It is done firstly by a discrete model and then by a continuous model that can be subjected to the method of integral in the future. Finally the proposed approach is validated by using a proved commercial simulation model to validate. This approach can be regarded as a simplified estimating method of traffic distribution and is also helpful for the researches on the road network density. We have to make clear that through our research we do not consider about the influence of congestion since our objective is limited to the planning stage only.

2. A Discrete Intersection Utilizing Model of a Single OD Pair

A road network consists of many links (roads) and nodes (intersections). For most modern urban road networks, the common knowledge is that the smoothness of traffic movement is more likely decided by intersections, rather than basic road sections. This research concentrates on the intersections, by assuming all origins and destinations of all *OD* trips are from or to the intersections, instead of the traditional zones. Similarly, routes between any *ODs* are also described by a sequence of intersections, rather than links.

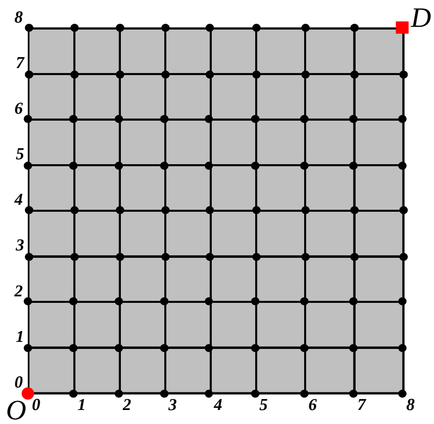

Figure 1: A Road Network Model

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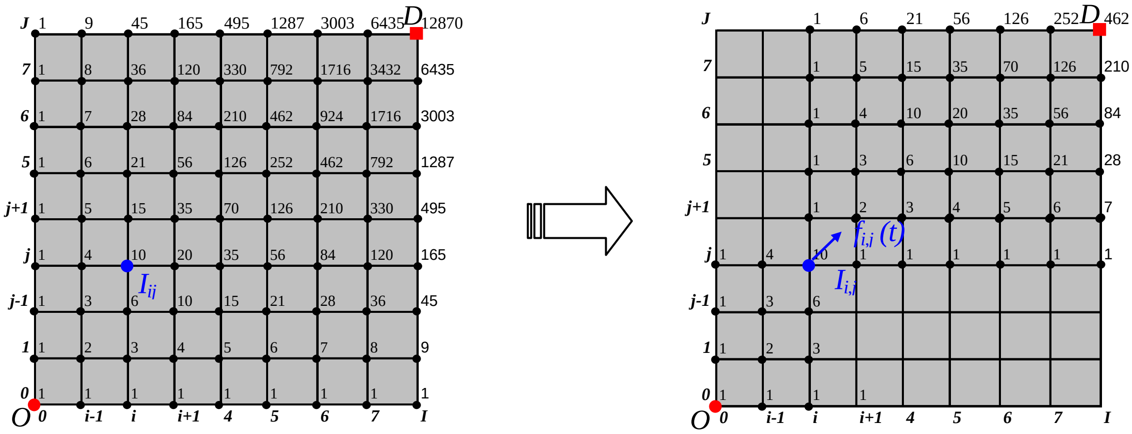


a) $R_o(i,j)$, route numbers from O b) Estimating Directional Traffics

Figure 2: An Intersection Choice Model

(1) Road Network Model

A road network that consists of intersections and the connecting roads is shown in Fig. 1. In this paper, we assume a road hierarchy[1)] with the high-hierarchical roads shown as solid lines on a uniform background of low-hierarchical roads (grey shade), as suggested by H. Lehmanh[2)].

(2) Discrete Intersection Choice Model for Estimating Directional Intersection Entering Traffics

As stated earlier, in this paper, we assume the percentage of traffic crossing a certain intersection equals to the utilizing probability of the intersection in the routes between an O and D.

a) Number of routes from O

Route choice is defined as choices of intersections in this paper, for the purpose to estimate directional intersection entering traffics. For a network of IJ with O and D at two opposite corners, the total number of route s from O to all the intersections are shown in Figure 2-a). If we define $R(O,I)$ as the number of routes from O to the intersection $I_{i,j}$, we can get the Equation 1.

$$R(O, I_{ij}) = \frac{[(i-0)+(j-0))]!}{(i-0)!(j-0)!} = \frac{(i+j)!}{i!\,j!} \tag{1}$$

b) Estimating Directional Traffics

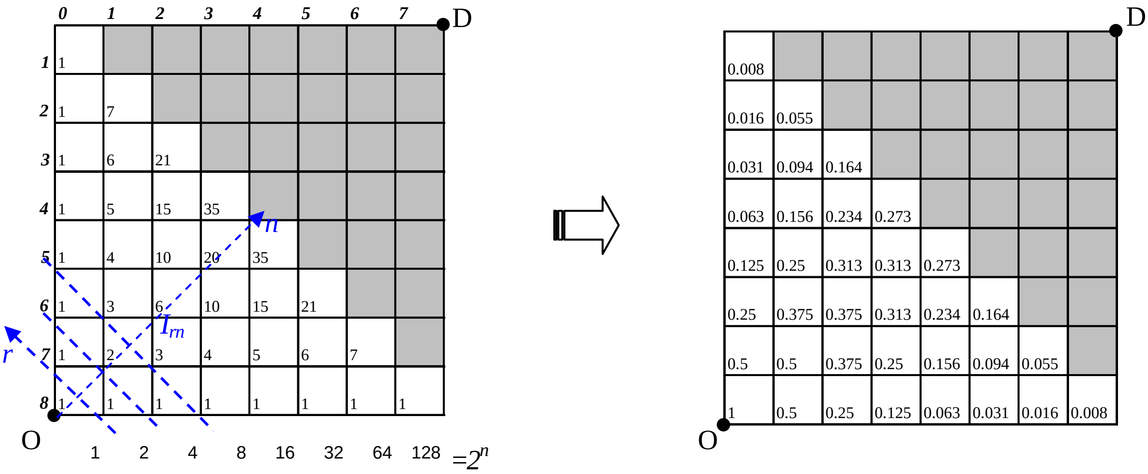
As shown in Figure 2-b), at the intersection $I_{i,j}$, the total entering traffic at time t, $f_{ij}(t)$, can be estimated by the following equation (2).

$$f_{ij}(t) = \rho_{OD}(t)\frac{R(O,I_{ij})\cdot R(I_{ij},D)}{R(O,D)} = \rho_{OD}(t)[\frac{(i+j)!}{i!\,j!}\cdot\frac{(I-i+J-j)!}{(I-i)!(J-j)!}/\frac{(I+J)!}{I!\,J!}] = \rho_{OD}(t)\cdot p_{ij} \tag{2}$$

where $R(Iij, D)$ is number of routes from Iij to D; $R(O,D)$ is the total number of routes from O to D; and $\rho_{OD}(t)$ is the trip generating/attracting rate between O and D.

- **Definition of Intersection Utilizing Probability (p_{ij})**,: here we define p_{ij} as the intersection utilizing probability from O to D. It can be calculated by the equation (3). Obviously, the value of p_{ij} depends on the network itself, thus independent from the traffic demand or the OD pattern.

$$p_{ij} = \frac{R(O,I_{ij})\cdot R(I_{ij},D)}{R(O,D)} = \frac{(i+j)!}{i!\,j!}\cdot\frac{(I-i+J-j)!}{(I-i)!(J-j)!}/\frac{(I+J)!}{I!\,J!} \tag{3}$$



a) Number of Routes b) Utilizing Probability

Figure 3: Intersection Utilizing Probability of a Symmetrical Network

3. Development of a Continuous Intersection Utilizing Model

Here, we try to develop a continuous model on the basis of the earlier discrete model under the condition of a symmetrical road network pattern. Such a symmetrical grid network is shown in Figure 4, the number of routes and the utilizing probability can be easily estimated by using the earlier model.

As for the new *r*-*n* coordinate system, when the traffic from O to D crossing the blue lines, p_{rn} the utilizing probability of an intersection I_{rn} on these blue lines can be calculated by the following equation (4) since it consist of a perfect binomial distribution.

$$p_{rn} = \frac{R(O,I_{rn})}{\sum_{r=0}^{r<=n} R(O,I_{rn})} = \frac{\binom{n}{r}}{\sum_{r=0}^{r<=n}\binom{n}{r}} = \binom{n}{r} / 2^n = \frac{n!}{r!(n-r)!}(\frac{1}{2})^n \tag{4}$$

where $R(O,I_{rn})$ is the number of routes from O to I_{rn}.

As $n \to \infty$, by assuming $x = \frac{r-n/2}{\sqrt{n}/2}$, and applying the Stirling Equation of $n! \cong \sqrt{2\pi} e^{-n} n^{n+1/2}$, an appropriate continuous equation can be obtained[3)].

$$p(r) \cong \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2}} \tag{5}$$

where $\sigma = \sqrt{n}/2$

4. Validation of the Proposed Model

In order to validate the proposed models, a 9 by 9 grid road network is constructed under the simulation package of SOUND/4U©, which is a macroscopic model developed under the cooperation between the Industrial Science Institute, University of Tokyo and i-Transport Lab. Co., Ltd (ITL).

For the purpose of comparison, assuming $\rho_{OD}(t) = 10000$, intersection entering traffics of the grid network are estimated by both equation (4) and the simulation package. The results can be found in the table 1 and table 2.

(1) Estimation Results

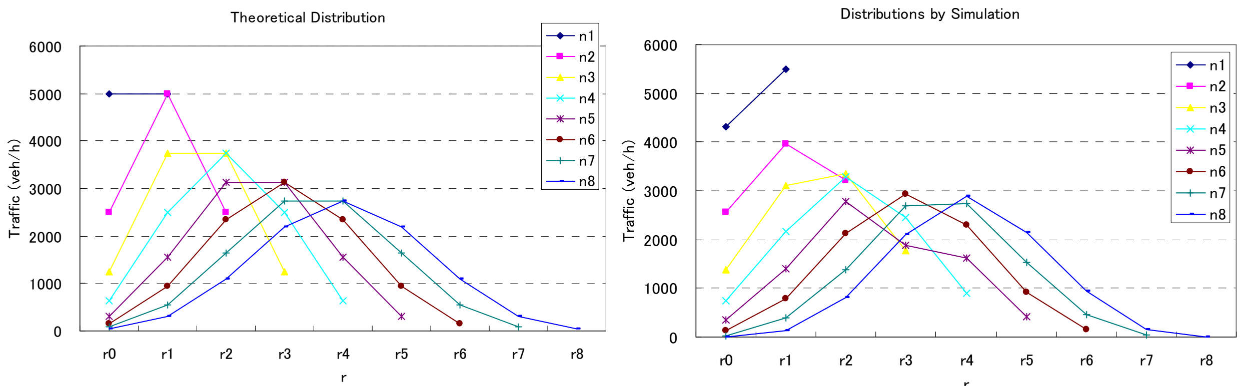
Table 1: Estimation Results of the Proposed Models

n \ r	0	1	2	3	4	5	6	7	8	Sum
0	10000									10000
1	5000	5000								10000
2	2500	5000	2500							10000
3	1250	3750	3750	1250						10000
4	625	2500	3750	2500	625					10000
5	313	1563	3125	3125	1563	313				10000
6	156	938	2344	3125	2344	938	156			10000
7	78	547	1641	2734	2734	1641	547	78		10000
8	39	313	1094	2188	2734	2188	1094	313	39	10000

Table 2: Estimation Results of SOUND

n \ r	0	1	2	3	4	5	6	7	8	Sum
0	9906									9906
1	4324	5491								9815
2	2553	3961	3209							9723
3	1386	3119	3344	1782						9631
4	735	2164	3295	2442	904					9540
5	343	1399	2778	1893	1627	408				8448
6	122	793	2128	2937	2301	919	156			9356
7	32	385	1381	2693	2727	1538	462	47		9265
8	6	121	811	2099	2893	2146	939	151	7	9166

(2) Traffic Distribution



a) Distribution by Proposed Model b) Distribution by Simulation

Figure 4: Comparison of Distributions of Intersection Entering Traffics

General speaking, the proposed model can give out an acceptable estimation result. Although for the simulation results, since many vehicles stay on the roads when the calculation finished, the further the intersections, the smaller the intersection entering traffics are estimated.

5. Concluding Remarks

In this research, an analytical approach is proposed to estimate intersection entering traffic distributions on a grid network. Both a discrete and a continuous model are developed. The estimation results are validated as acceptable by a mature commercial simulation platform. The proposed approach is a totally analytical model that applying network parameters only. Such kinds of approaches are especially important to the planning stage when detailed OD data is not available and the planners have the need to fully understand relationship between the network parameters and the traffic distributions. Its influence on both the academic and practical fields is expected.

In future, the author will concentrate on two parts: 1) the case of a full All To All OD distribution, and 2) the case of the non grid networks. The major purpose of the future research will also be consistent, to describe a network using the network parameters only.

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